Bank asset structure and the risk-taking implications of capital and liquidity requirements

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Abstract

In addition to risky loans, banks hold risky securities that provide uncertain future liquidity. This leads them to choose an asset structure with their desired correlation between liquidity and long term asset returns. We show that liquidity management and risk management concerns lead to a trade-off that creates an inverse relationship between security holdings and aggregate asset risk. Capital requirements mitigate liquidity risk in all future states of the world, thereby reducing the cost of liquidity risk and leading banks to increase aggregate asset risk. Liquidity requirements such as the Liquidity Coverage Ratio (LCR) affect high liquidity shock states and mitigate aggregate asset risk-taking. These results highlight the tension between capital and liquidity regulations in addressing the risk taking incentives of financial intermediaries.

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In the 1950s liquid assets were typically 30 percent of British clearing banks total assets, and these largely consisted of Treasury Bills and cash. At June 2007, cash is about 1/2 *per cent and traditional liquidity about 1 per cent of total liabilities.*

- Tim Congdon, *Financial Times - 2007*

1 Introduction

The academic literature on banking regulation has traditionally focused on the liability side of bank balance sheets while theories related to banking assets have generally focused on the origination, monitoring, and sale of loans as well as portfolio choice.[1] The regulatory implications of the staggering transformation of banking assets over the past five decades have been relatively overlooked: While the percentage of safe, liquid assets remained fairly stable, risky securities have become an important part of banking assets. At the end of 2012, more than 50% of the securities held by US commercial bank consisted of risky securities like MBS, CMO, CMBS, corporate and municipal bonds, and other ABS (Hanson et al. (2015)). Do these risky security holdings represent a threat to financial stability?

This paper examines how this transformation of bank asset structure has affected risk taking and discusses the interplay between bank asset allocation and risk taking, on the one hand, and capital requirements and and the liquidity coverage ratio, on the other hand. In contrast to traditional banking models that examine liquidity risk, we allow banks to allocate their assets optimally between risky loans and risky securities. Most existing models assume that bank securities are perfectly safe, akin to cash or perfectly safe government bonds. Introducing risky securities gives rise to a specific type of risk that has been neglected in the literature: the correlation between illiquid and liquid assets. As this correlation represents a risk on the overall portfolios of loans (illiquid assets) and securities (liquid assets), we will refer to it as *aggregate asset risk*. Interestingly, it can also be viewed

[1] See, among many others, Diamond and Dybvig (1983), Diamond (1984), Gorton and Pennacchi (1995), Cerasi and Daltung (2000), Freixas and Rochet (2008), Parlour and Plantin (2008), Acharya and Schnabl (2009), Hartman-Glaser, Piskorski, and Tchistyi (2012), Chemla and Hennessy (2014)

as a "liquidity" *wrong-way risk* in that it captures the risk that the liquidity of securities deteriorates at the same time as the value – or creditworthiness – of the loans. As in the traditional definition of wrong-way risk – the risk that the credit exposure of counterparty A to counterparty B increases at the same time that the creditworthiness of Counterparty B deteriorates – this risk can be specific or general. Specific aggregate asset risk can arise because security returns are fundamentally correlated with loan returns. For instance, a bank is exposed to specific aggregate asset risk if it holds mortgage loans along with mortgage backed securities. General aggregate asset risk can come from macroeconomic factors that affect both the creditworthiness of loans and securities liquidity, such as market freezes events observed during the 2007-2008 financial crisis.[2]

Interestingly, regulators do appear to recognize aggregate asset risk in their definitioin of high quality liquid assets. In the latest framework of bank supervision, known as Basel III, the Basel Committee introduced a new set of liquidity regulations with two main new ratios: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). The ratio of interest in this study, the LCR, is defined as the amount of High Quality Liquid Assets (HQLA) over the total net cash outflow over the next 30 days. Among the four fundamental characteristics that make an asset a HQLA, one is that it needs to have a "low correlation with risky assets" (Basel Committee (2013), p.13). However, while the existence of a correlation between liquidity and long term asset returns appears to be recognized, the Basel III framework is unclear on how the combined use of capital or liquidity requirements affects bank risk taking. Instead, it is generally claimed that risk is taken care of during the process of what is called "stress testing", where several predefined scenarios should reveal worrying correlations in bank balance sheets. This paper formally analyses this risk and sheds some lights on how capital and liquidity requirements affect the incentives of banks to take on aggregate asset risk. We show that capital requirements reduce aggregate asset risk while liquidity requirements reduce aggregate risk-shifting.

In the model, a bank can invest in long term assets – or loans – and securities. Loans can be thought of as corporate loans that cannot easily be securitized and are fundamen-

[2]See Gorton (2010).

tally illiquid because they cannot be liquidated without a loss at the time of a liquidity need. Securities are risky in that they provide uncertain future liquidity. This assumption can be viewed from different angles. Securities can be seen as perfectly liquid but providing uncertain future returns. It is the case of risky asset backed securities in a perfectly rational market where investors have full information about future securities returns. Uncertain future liquidity can also capture time varying liquidity. A specific characteristic of liquidity is that it is not only asset dependent but also time dependent. Securities that are liquid in good times can suddenly become illiquid due to market conditions such as investors sentiments. Regardless of the interpretation, it seems fairly reasonable to assume that securities do not always provide liquidity in every possible future states of the world. This is perhaps even more relevant considering today's debate about the scarcity of safe assets, as underlined by the IMF in its Global Financial Stability Report (2012, chapter 3). The bank's endogeneous choice of aggregate asset risk is subject to two conflicting forces stemming from liquidity management and risk management motives. The bank is subject to a stochastic liquidity shock on its liabilities due to uncertain withdrawals from depositors, as in Diamond and Dybvig (1983). Liquidity management aims at limiting the risk of liquidity shortage at the time of deposit withdrawals. Because the bank is subject to limited liability in the worst state of the world, it has incentives to correlate loan returns with securities liquidity in order to maximize future expected returns. Liquidity risk creates aggregate risk-shifting. On the other hand the bank engages in risk-management and wants to minimize returns volatility, as in a traditional portfolio analysis. This limits the correlation induced by liquidity risk and gives rise to an equilibrium choice of aggregate asset risk. Risk management arises from the cost associated with liquidity shortages and captured by firesales. Because the model introduces risky securities that can act as a liquidity buffer, it provides the opportunity to study interactions between liquidity management and risk management, something that cannot been analysed with a traditional portfolio framework – focusing on risk management – or liquidity models – focusing on liquidity risk.

I show that capital and liquidity requirements have two opposite effects on bank incentives because they have different effects in different future states of the world. Both regulations limit liquidity shortages but they differ in their contingency. The effects of capital requirements are state independent and affect all future states of the world. It thus reduces risk management concerns and incentivizes the bank to increase aggregate asset risk. On the other hand, liquidity requirements provide state dependent liquidity thus decreasing the effects of limited liability on risk-loving incentives. Similarly, by imposing a minimum amount of securities to be held by the bank, liquidity requirements provide liquidity in the states where securities are the most liquid. Therefore their impact on the bank is similar to an increase in correlation. Hence, the resulting choice of aggregate asset risk decreases. In other words, liquidity requirements provide desirable liquidity characteristics that would have otherwise been created by an increase in aggregate asset risk. The state independent characteristic of capital requirements fails to capture this effect.

Section 2 provides a brief review of the literature. Section 3 layouts the model and the results. Section 5 concludes.

2 Literature

Since Diamond and Dybvig (1983), liquidity risk has been the focus of many studies but most of the literature uses the simplifying assumption that banks only hold cash as liquid securities. We take a novel approach by extending the set of liquid securities available to the bank and by assuming that they can invest in liquid but risky securities. Assuming that liquid securities are perfectly safe is sufficient for the large part of the banking literature on liquidity that focuses on the role of banks in liquidity provision and liquidity transformation. Diamond and Dybvig (1983) explain how banks can provide liquidity to households while investing in long-term illiquid projects and their model argues in favor of deposit insurance to prevent costly bank runs. Diamond and Rajan (2001) go one step further in understanding why long-term illiquid assets may be coupled with fragile liabilities such as demand deposits. They show that the fragility of bank liabilities disciplines the banks and enhance the value of long-term illiquid assets. The fact that banks are prone to liquidity problems leads Kashyap, Rajan, and Stein (2002) to show that banks can economize on costly liquidity buffers by holding assets and liabilities with imperfectly correlated liquidity risk. The liquidity creation role and run prone characteristic play an essential part in understanding the interactions between assets and liabilities of financial intermediaries but these theories do not offer much insight about the optimal portfolio allocation and risk-shifting[3].

The absence of the modeling of risky securities is also due to a lack of rationale for banks to hold marketeable securities. Hanson et al. (2015) note that today's commercial banks are mainly funded with safe deposits but invest in risky loans and risky securities. They provide a possible explanation of why commercial banks are holding long-term illiquid loans and risky securities. A safe deposit structure is alike the liability focused view of banking started by Gorton and Pennacchi (1990) and arguing for a "narrow banking" system where the fundamental role of financial intermediaries is to create safe-like securities for depositors[4]. In Hanson et al. (2015), commercial banks and shadow banks are competing in the business of creating safe-like securities but they do so differently depending on their funding structures. Traditional banks are assumed to have a stable source of funding that gives them a comparative advantage at holding illiquid loans and risky securities. On the contrary, shadow banks are subject to runs and firesales losses and are more likely to hold safe and short-term assets to back their liabilities. While the funding structure is most certainly an important determinant of the asset structure of financial intermediaries, it does not take into account two important functions that commercial banks perform, namely liquidity management and risk management.

While simplistic in essence, viewing banks as holding risky illiquid and liquid assets raises basic yet fundamental questions. The one that is the focus of this paper is to understand the risk behavior on the overall bank's balance sheet, that is the aggregate portfolio choice of illiquid and liquid securities, and more specifically the aggregate correlation between illiquid and liquid assets. This question is very much like the traditional consid-

[3] Repullo (2005) is a counter example with cash and endogeneous risk-shifting on the long term asset. Recently, Calomiris, Heider, and Hoerova (2015) analyse risk management for a bank with cash holdings.

[4] See also Pennacchi (2012)

erations of portfolio management. Indeed, considering banks as portfolio managers dates back to Pyle (1971) and Hart and Jaffee (1974). The portfolio approach explicitly considers the risk management performed by financial intermediaries by building on the classical mean-variance portfolio approach of Markowitz (1952). Hart and Jaffee (1974) show the existence of a separation theorem when the portfolio approach is applied to banks. That is, the mix of assets chosen by financial intermediaries is independent of the parameters of the utility function and can be logically separated from the decision on the size of the portfolio. This approach is the natural tool to study the effects of capital requirement on risk shifting. This has been done, among others, by Koehn and Santomero (1980), Kim and Santomero (1988) and Rochet (1992). Kim and Santomero (1988) shows that an increase in capital requirements does not necessarily decrease a bank's probability of failure because of portfolio reshuffling. Kim and Santomero (1988) argues that risk-weighted capital requirements must be used if one wants such regulation to be effective, and Rochet (1992) shows that capital regulations are effective only when banks are behaving as portfolio managers as opposed to value maximizers, highlighting again the need for a risk-weighted approach.

One weakness of the portfolio approach is that it fails to capture assets heterogeneity in their liquidity dimension. As a result, studies focusing on liquidity risk as has been using a simpler approach, while abstracting from standard portfolio decisions, such as correlations[5]. In a sense this paper lies in between liquidity management and risk management by sheding lights on portfolio choices and risk-shifting coming from liquidity risk. It extends the traditional models of liquidity risk in the vein of Diamond and Dybvig (1983) by relaxing the bank's set of liquid assets and introducing the choice of correlation between illiquid and liquid assets. By doing so, it analyses a type of risk referred to as *aggregate asset risk* that has not been formally studied before.

[5]Acharya and Pedersen (2005) provide an asset pricing model considering a different type of liquidity risk: the risk of not being able to easily sell a security. I study here liquidity risk coming from liabilities. In the taxonomy of Brunnermeier and Pedersen (2009), I focus on funding liquidity as opposed to market liquidity.

3 The Model

The model features an economy with three dates $t \in \{0, 1, 2\}$, one good, and two types of agents: banks and households.

Households There is a continuum of households of size one, each endowed with one unit of good at time 0 that can be consumed at time 1 or 2. As in Diamond and Dybvig (1983), each household faces the risk of being an early or a late consumer. There is aggregate uncertainty[6] in the economy represented by a state of nature s that can take two values, H and L, with equal probabilities. The probability of being an early consumer in state s is denoted by λ_s where

$$0 < \lambda_L < \lambda_H \leq 1$$

Aggregate uncertainty is resolved at date 1 when the state s is publicly observed. We denote p_s the probability of state s to realize. The discount rate is normalized to 0 and households are risk neutral with utility

$$u_s(c_1, c_2) = \begin{cases} c_1 \text{ w.pr. } \lambda_s \\ c_2 \text{ w.pr. } (1 - \lambda_s) \end{cases}$$

Where c_t represents consumption at date t. There exists investment opportunities in the economy but we assume that households do not have the necessary skills to undertake them. Rather, financial intermediation is essential in that households invest their endowment in a bank that undertakes investment opportunities on their behalf.

Banks There is one bank that is assumed to be representative of the banking sector. The bank has no initial endowement and issues financial claims to households. It issues short term demand deposits that can be claimed by households at any time 1 or 2 as well as equity capital. We assume that deposits are ensured so that households do not require a risk-premium for holding risky deposits. Because the banking sector is the only intermediary in

[6]Aggregate uncertainty is as in Allen and Gale (2007).

the economy, it absorbs all households initial endowements. The overall size of the banking sector is thus one. The bank's capital structure is exogeneously fixed with k in capital and $(1-k)$ in deposits. An exogeneous capital structure is a strong assumption. It is true if any capital ratio imposed by a regulatory body is binding in equilibrium, which is assumed here. This assumption prevents us to understand the rationale of banking regulation that has to be taken as given. Among others, Allen and Gale (2007) and Rochet (1992) analyse this question. The model can be generalized by introducing short term debt along with deposits as liabilities. The results go through as long as there is uncertainty on the amount of liquidities needed at the interim date, time 1. In this model, the stochasticity of the liquidity shock comes from uncertain deposits withdrawals but it can arise from short-term debt combined with uncertain access to refinancing.

At time 0, the bank provides firm lending. Loans are risky and return R or 0 with equal probabilities. Without loss of generality, we assume that all loans are perfectly correlated ; they either all return R or all return 0. Loans are illiquid in the sense that the bank cannot sell them on the market without incurring firesale losses that will be specified later.

At time 1, aggregate uncertainty about household preferences is resolved and a fraction of depositors withdraw their deposits. It is assumed that external financing is infinetely costly for banks at that date. Therefore, the bank can only rely on existing assets to face depositors withdrawals. If the bank only holds firms loans, it is subject to firesales losses. To prevent this adverse effect, the bank can invest in assets that provide liquidity at the interim date. We denote them *securities*. The focus of the paper is to study an economy in which perfectly safe assets are not available. It means that the bank cannot hold assets that are providing liquidity in every future state of the world. Therefore we assume that securities provide either r or 0 units of liquidity at the interim date with equal probabilities. This assumption can be interpreted in two ways. First, one may think of securities as being easily marketeable assets that provide uncertain returns, such as risky securitized assets. Even if these assets can be sold at their fundamental value, their ability to generate liquidity is state dependent because of their specific risk. Second, the ability for an asset to provide liquidity does not only depend on the asset alone, but also on the macroeconomic environment or the willingness of economic agents to trade. For instance, the value of government bonds that can be considered safe depends on monetary policy and interest rates. Also, market sentiment can quickly turn a liquid security into a very illiquid asset. One can think of episodes of liquidity dry ups during the 2007-2008 financial crisis. The key point is that it is very difficult for economic agents such as banks to know with certainty the future ability of assets to provide liquidity, which by construction makes liquidity uncertain. Here, we assume that future liquidity is stochastic but that its distribution is known ; there is no ambiguity about the liquidity that securities provide in future periods. Also, we assume that the bank and market participants behave rationally. Assuming that perfectly safe assets are non-existent implicitely assumes markets incompleteness. However, even if markets are complete, it can be shown that banks optimally use risky securities as long as they are sufficiently cheaper than safe assets. This condition is likely to be true in today's economic environment in which safest assets are in the negative rates territory.

At time 0, the bank chooses the amount to invest in loans and liquid assets. It lends $(1-\gamma)$ to firms and buys γ securities. By construction, the bank cannot choose the individual risk of securities and loans. Instead, it faces a panel of borrowing firms and needs to choose how much to lend to each firm. This flexibility allows the bank to adjust the correlation between the returns of the loans portfolio and the liquidity provided by the securities. Assume the bank holds a portoflio of securities backed by real estate assets and that it can lend to either a construction or an agricultural firm. The risk on the real estate markets creates uncertainty on future liquidity provided by the securities. It is likely that the creditworthiness of the construction firm is positively correlated with securities liquidity. However the agricultural firm is not impacted by real estate market uncertainty. By choosing between lending to the construction or to the agricultural firm the bank can influence how its access to liquidity covaries with future expected loan returns. Instead of modelling the underlying portfolio choice with a set of securities and firms, we express the problem in a reduced form, and tie securities and loans with an endogeneously chosen correlation. The probability of loans to return R conditional on securities providing r units of liquidities is given by:

$$P[\tilde{R} = R | \tilde{r} = r] = \frac{1}{2}(1 + \rho)$$

Where ρ is a proxy of the correlation between loan returns and securities liquidity. For simplicity we focus on positive correlations and assume

$$0 < \rho < 1$$

For the problem to be interesting, there needs to be a cost of holding securities. We assume that investing in long term real assets is more profitable than holding liquidities:

$$R > r$$

At time 1, the bank is subject to a liquidity shock on its deposits. It faces withdrawals of a fraction λ_H of its deposits with probability p_H, and λ_L with probability $p_L = 1 - p_H$. Because securities can be sold at their fundamental values the bank uses them in priority to face the liquidity shock. If not enough liquidity is available through the sale of securities, it is forced to sell long term loans on the market. Securities liquidity and loan returns are covarying so the value of the loans are impacted by the state of the economy as well as the ex-ante choice of correlation. If securities end up providing low liquidity and loan returns are highly correlated with liquidities, their value is low and the bank needs to sell a large amount of them to face the liquidity shock. The bank defaults if it cannot raise enough liquidity by selling securities and loans combined. Otherwise it carries the remaining loans to time 2. The bank maximizes shareholders value, that is time 2 equity value. Figure 1 represents the timeline of the model as well as the bank's balance sheet structure.

Firesales We assume that there is a price impact of selling loans on the market. The more loans the bank needs to sell, the lower their unit price. It captures the endogeneity of firesale prices as in the models of Shleifer and Vishny (1992), Allen and Gale (1994), Diamond and Rajan (2011) or Stein (2012). Firesale are such that if the bank wants to raise x units of liquidity, it needs to sell $F(x)$ worth of time 2 expected loan returns. We

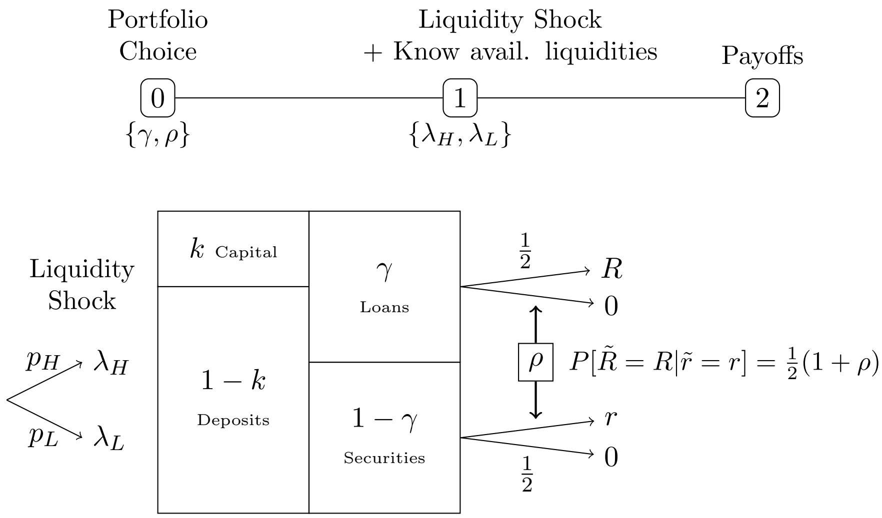


Figure 1: Setup

make the following assumption regarding the firesales.

Assumption 1.

$$F(x), F'(x), F''(x) > 0 \tag{1}$$

$$F(0) = 0 \tag{2}$$

$$F'(0) > r \tag{3}$$

Assumption 1 states that F is positive, increasing and strictly convex. Moreover, it forces firesales to be costly enough so that it is always suboptimal to use illiquid assets instead of liquid securities as liquidity provider.

A linear F is akin an exogeneous firesale discount independant of the number of sold illiquid assets. Price impact is captured by forcing the strict convexity of F and price impact losses are captured by the following expression:

$$L(x) = xF'(x) - F(x)$$

$L(x)$ represents the loss due solely to price impact. A linear F would result in $L(x) = 0$. Note that the price impact loss is strictly increasing in the amount of loans sold as $L'(x) =$

$F'(x) > 0$.

Intuitions Before diving into the resolution, it seems important to describe the general mechanisms behind the results. The focus of the paper is to understand the endogeneous choice of aggregate asset risk as defined above, that is, the correlation between the portfolios of illiquid loans and liquid securities. To that end, we try to capture important determinants of this choice of risk and denote them liquidity management and risk management. Liquidity management aims at supporting long term assets[7], or loans, as well as avoiding liquidity shortages. Because the bank is subject to limited liability, liquidity management gives incentives for the bank to correlate liquidity with loan returns and thus increase aggregate asset risk. Indeed, it is profitable for the bank to support long term assets when these assets are the most valuable. It incentivizes the bank to secure more liquidity in the states in which loans have high returns. The effect is much like standard risk-shifting except that it arises from liquidity considerations and not purely from capital structure. In fact we will see that unlike traditional risk-shifting, aggregate asset risk decreases with leverage. Risk management is usually aiming at reducing ex-post variance in returns, as in Froot, Scharfstein, and Stein (1993) or Froot and Stein (1998). In our context, the goal of risk management is to mitigate the cost of liquidity shortages that are embedded in firesale costs. While captured differently, the effects are identical are reducing firesales losses is attained by lowering the variance between liquidity provisions and loan returns. To decrease firesales losses, the bank tries to enhance loans value when liquidity is scarce. This is achieved by decreasing the correlation between loan returns and liquidity provision. Risk management in this model incentivizes the bank to decrease aggregate asset risk. It is the trade-off between these two forces that gives rise to an endogeneous choice of aggregate asset risk. Both channels are illustrated in figure 2.

Now that we described all agents and the firesales mechanism, we can analyze the bank choices. We solve the model by backward induction. All proofs are provided in the appendix.

[7] As in Holmstrm and Tirole (2011)

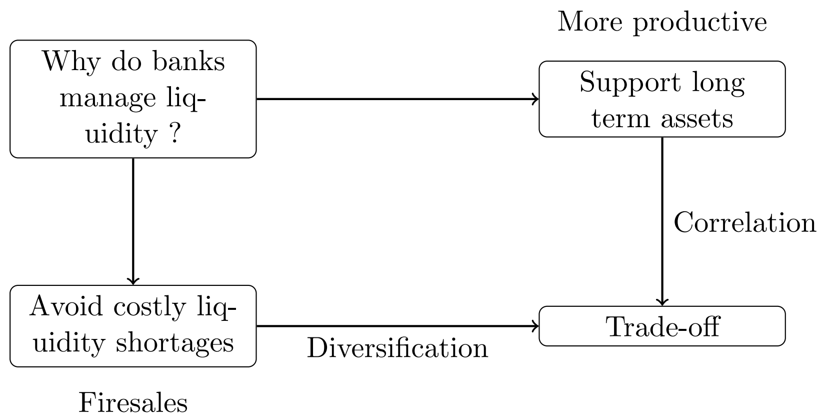


Figure 2: Trade-off

3.1 Liquidity shock

At time 1, the bank holds a fraction γ of securities and needs to pay $\lambda_s(1-k)$ to early depositors, where s relates to a high (λ_H) of low (λ_L) liquidity shock. Securities are natural providers of liquidity and are first used to pay depositors. If not enough liquidity is available through securities, the bank sells loans on the market and is subject to a price impact loss from firesales.

When securities provide r units of liquidity, the bank can repay up to γr to depositors at no cost. More early depositors means the bank needs to raise an additional $\lambda_s(1-k)-\gamma r$ units of liquidity by selling illiquid loans. It can either have enough loans and pay back all depositors at time 1 or default. If liquidity is scarce ($\tilde{r}=0$), the bank has to raise the full $\lambda_s(1-k)$ through loans sales. Lemma 1 defines the thresholds of early depositors at which the bank is subject to firesales or defaults.

Lemma 1. *There exists $\lambda_0 < \lambda_2$ and $\lambda_1 < \lambda_2$, such that, for a given liquidity shock state s,*

- *When securities provide r units of liquidity:*
  - *If $\lambda_s < \lambda_0$, the bank has enough liquidity to face withdrawals from depositors and there are no firesales.*
  - *If $\lambda_0 < \lambda_s < \lambda_2$, the bank needs to sell an amount $F(M_r^s)$ of loans.*

– *If* $\lambda_s > \lambda_2$*, the bank defaults.*

Where $M_r^s = \frac{\lambda_s(1-k)-\gamma r}{\frac{1}{2}(1+\rho)R}$

- *When securities provide no liquidity:*

  – *If* $\lambda_s < \lambda_1$*, the bank needs to sell an amount* $F(M_0^s)$ *of loans.*

  – *If* $\lambda_s > \lambda_1$*, the bank defaults.*

Where $M_0^s = \frac{\lambda_s(1-k)}{\frac{1}{2}(1-\rho)R}$

When securities provide r units of liquidity, the expected value of one unit of loan is $\frac{1}{2}(1+\rho)R$. The bank needs to sell an amount $\max\left\{0, F\left(\frac{\lambda_s(1-k)-\gamma r}{\frac{1}{2}(1+\rho)R}\right)\right\}$ of loans to obtain extra liquidity. The bank defaults if $(1-\gamma) < F\left(\frac{\lambda_s(1-k)-\gamma r}{\frac{1}{2}(1+\rho)R}\right)$, defining the threshold λ_2. If securities provide no liquidity, the expected value of one unit of loan is $\frac{1}{2}(1-\rho)R$ and the bank has to sell $F\left(\frac{\lambda_s(1-k)}{\frac{1}{2}(1-\rho)R}\right)$ units of loans. The bank defaults if $(1-\gamma) < F\left(\frac{\lambda_s(1-k)}{\frac{1}{2}(1-\rho)R}\right)$, defining the threshold λ_1. It is trivial to see that $\lambda_1 < \lambda_2$ as long as ρ is large enough, and it is always true when $\rho > 0$.

We assume that the bank does not always default when liquidity is scarce. That is, the low liquidity shock is low enough so that the bank can survive by selling illiquid loans.

Assumption 2.

$$\lambda_L < \lambda_1$$

We can now turn to time 0 portfolio choices.

3.2 Portfolio choice

At time 0, the bank needs to choose both the investment mix between liquid and illiquid assets, γ, and the correlation between the two, ρ, also referred to as *aggregate asset risk*.

Before laying out the bank's objective function, we can restrict the set of acceptable choices for γ. Holding liquidity is costly because illiquid assets are more productive. It is thus suboptimal for the bank to hold more liquid assets than what is necessary to face the highest liquidity shock λ_H. Equivalently, liquid securities allow the bank to carry illiquid assets to maturity. Using illiquid loans as providers of liquidity is always more costly than using securities, as is ensured by assumption 1. Therefore it is suboptimal to use illiquid loans as liquidity buffer at time 1, and the bank must at least hold enough liquid assets to face the lowest liquidity shock λ_L.

Lemma 2 formalizes these results.

Lemma 2. *The optimal mix of assets $\gamma*$ is such that*

$$\underline{\gamma} \equiv \frac{\lambda_L(1-k)}{r} \leq \gamma* \leq \frac{\lambda_H(1-k)}{r} \equiv \overline{\gamma}$$

which is equivalently expressed as

$$\lambda_L \leq \lambda_0 \leq \lambda_H$$

It is important to note that the thresholds $\lambda_0, \lambda_1, \lambda_2$ defined in lemma 1 are dependent upon the bank choices. In fact, it is reasonable to expect that the ability of the bank to impact its default probabilities is an important determining factor of the choice of γ and ρ. We now show that the bank *endogeneously* defaults in the worst state of the worls. At the optimum, the bank choices are such that it always defaults in the high shock state if liquidity is scarce, and it never defaults if securities provide liquidity. This is formalized in lemma 3.

Lemma 3. *The optimum bank's portfolio is such that*

$$\lambda_1 < \lambda_H < \lambda_2$$

This result is central for the mechanisms at play as it creates a non null *endogeneous* probability that the bank defaults if securities do not provide liquidity. Combined with limited liability, it creates risk-shifting incentives.

We can now express the bank's objective function. The expected value of time 2 bank's

	Low shock (λ_L, p_L)	High shock (λ_H, p_H)
High Liq. $(\tilde{r} = r, \frac{1}{2})$	Enough Liquidity No Firesales	Firesales Sell for M_r^H in loans
Low Liq. $(\tilde{r} = 0, \frac{1}{2})$	Firesales Sell for M_0^L in loans	Default

Figure 3: States of the world at time 1

equity, denoted V, is given by

$$\begin{aligned} V = p_L &\left\{ \gamma r + (1-\gamma)\frac{1}{2}(1+\rho)R - \lambda_L(1-k) + [(1-\gamma) - F(M_0^L)]\frac{1}{2}(1-\rho)R \right\} \\ &+ p_H \left\{ [(1-\gamma) - F(M_r^H)]\frac{1}{2}(1+\rho)R \right\} \\ &- p_L(1-\lambda_L)(1-k) - p_H(1-\lambda_H)(1-k) \end{aligned} \tag{4}$$

Where M_0^L and M_r^H are the amounts of loans that need to be sold in states where securities do not provide liquidity and when the liquidity shock is low (M_0^L), or when securities provide liquidity and when the liquidity shock is high (M_r^H).

The first line corresponds to the expected equity value in the low liquidity shock state. When securities provide liquidity (3 first terms), there are no firesales, the bank has enough liquidity to pay depositors, and all illiquid assets ($(1-\gamma)$) are carried up to time 2. When securities do not provide liquidity (last term), the bank needs to raise M_0^L by selling illiquid loans. The second line is the equity value in the high liquidity shock state. By lemma 3, the bank defaults if liquidity is scarce. Otherwise it needs to sell M_r^H units of loans to pay early depositors. The last line is simply the payment of late consumers at time 2. Figure 3 illustrates the different states of the world at time 1.

Note that if the bank does not default at time 1, it is assumed to pay back late depositors in full. This is not realistic if loan returns are insufficient in the last period. Apart from simplicity, this assumption ensures that limited liability in the last period does not affect the results. We are interested in the effects of the liquidity shock alone, and limited liability at time 1 is at the root of risk-shifting. However, we do not aim at capturing the effects of limited liability in the last period. Traditionally, leverage affects risk-shifting on the loans portfolio if there is limited liability at time 2. This effect is of no interest here. In fact the model does not allow to capture risk-shifting on the loans portfolio because risk is fixed on individual portfolios by assumption. Hence, removing the limited liability assumption at time 2 does not affect the generality of the results.

Risk-Taking trade-off The value function clearly shows the risk-taking trade-off. It can be broken down into risk-loving incentives in the high shock state and diversification incentives in the low shock state.

In the high liquidity shock state, the bank does not default only if securities provide liquidity ($\tilde{r} = r$). Limited liability in that state creates risk-loving incentives, and the bank can increase time 2 equity value by holding illiquid assets whose returns are correlated with its securities. Three effects are at play: increasing correlation allows the bank to sell fewer loans, reduces the price impact loss, and increases loan expected returns. A similar mechanism appears in the low liquidity shock state where the bank incurs firesales only when securities do not provide liquidity. Decreasing the correlation reduces the firesales losses by decreasing the amount of loans to sell, reducing the price impact loss, and increasing expected returns. This trade-off only appears because firesales can occur in both the high and the low shock state. Therefore, normalizing λ_L to zero would remove the diversification incentives and result in a correlation always at 1 [8].

The bank optimizes expected equity value by choosing the mix of assets as well as the correlation between the portfolios of loans and securities:

$$\max_{\gamma,\rho} V(\gamma, \rho) \tag{5}$$

$$\text{s.t. } \underline{\gamma} < \gamma < \overline{\gamma} \tag{6}$$

Where V is as in equation 4 and inequation 6 comes lemma 2. Lemma 4 characterizes

[8] This is because diversification incentives are only captured by firesales. One could think of a model where diversification comes from a time 2 concave investment opportunity.

the solution.

Lemma 4. *The optimal choices of mix of assets γ^* and correlation ρ^* follow the following equations:*

$$\gamma^* = \max\left[\underline{\gamma}, \overline{\gamma} - \frac{1}{2r}(1+\rho^*)RF'^{-1}\left(\frac{1}{r}\left[\frac{1}{2}(1+\rho^*)R + (R-r)\frac{p_L}{p_H}\right]\right)\right] \tag{7}$$

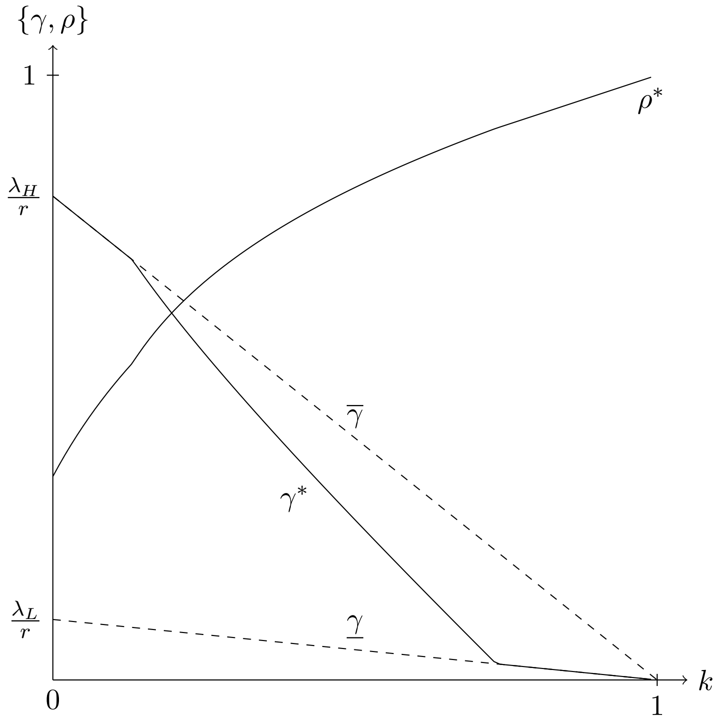
$$p_H(1-\gamma^*) + [p_H L(M_r^H(\gamma^*, \rho^*)) - p_L L(M_0^L(\rho^*))] = 0 \tag{8}$$

Where $L(x) = xF'(x) - F(x)$

Lemma 4 reveals that the risk-taking trade-off described previously creates a trade-off between the mix of assets and correlation that is at the basis of the subsequent results on regulatory implications. Equation 7 is the first order condition for γ. It shows that optimal security holdings are inversely related to aggregate asset risk. That is, the higher the correlation between the portfolios of loans and securities, the lower the amount of securities held by the bank. The first order condition for ρ (equation 8) shows an identical relationship. To see it, note that M_r^H is decreasing in both ρ and γ and M_0^L is increasing in ρ. Therefore, any decrease in γ^* has to be compensated by an increase in ρ^*. The intuitions are the following. On the one hand, when aggregate asset risk is higher, gains for holding loans are increasing because it increases expected returns when securities provide liquidity. Expected returns decrease when liquidity is scarce in the low shock state, but can be compensated by holding more loans. On the other hand, when the bank holds more securities, it reduces firesales losses in the high shock state but not in the low shock state. One way to compensate the loss in the low shock state is to increase the correlation thus lowering firesales in that state.

Lemma 5. *aggregate asset risk and securities holdings are inversely related.*

Let us illustrate the bank's optimal choices by numerically solving the model. We use the functional form $F(x) = e^{rx} - 1$ for the firesales. Figure 4 plots the optimal mix of assest γ^* and correlation between loans and securities ρ^* as a function of the leverage. It clearly shows the trade-off between the mix of assets and aggregate asset risk.



The firesale function is $F(x) = e^{rx} - 1$ and the parameters are $p_L = .8, p_H = .2, R = 1.05, r = 1, \lambda_L = .1, \lambda_H = .8$.

Figure 4: Optimal mix of assets and correlation as a function of leverage.

We have seen that liquidity risk combined with firesales create a risk-taking trade-off. Moreover, this trade-off leads to an inverse relationship between security holdings and aggregate asset risk taking. Next section draws regulatory implications for two of the most important tools used in banking regulation that are capital and liquidity requirements.

4 Regulatory Implications

This model is a partial equilibrium approach of optimal portfolio holdings. As it does not formally include a regulator with a social objective function, it stays silent on the rationale for regulation as well as on optimal regulation. Nevertheless, by shedding lights on how banks manage their assets when facing liquidity risk, it allows us to understand how banks respond to regulatory changes. We focus our analysis on capital and liquidity requirements.

Figure 4 paves the way for intuitions. It first shows that a bank with a higher capital ratio has a higher equilibrium aggregate asset risk. It also illustrates the inverse relationship between securities holding and aggregate asset risk which says that forcing banks to hold more liquid assets decreases aggregate asset risk. Finally, it suggests that the sensitivity of the effects of liquidity requirements on aggregate asset risk is higher for more levered banks. That is, ρ decreases more as a result of an increase of γ for low values of k. We analyse in turn these intuitions.

4.1 Capital Regulation

Historically, the main tool in micro and macro prudential regulations is the use of capital requirements, introduced in Basel I and Basel II regulatory frameworks. Lowering the leverage of financial intermediaries aims at reducing their default probability. Additionally, traditional risk-shifting mechanism suggests that more levered institutions have an accute appetite for risk, which may not be socially optimal. This is true in traditional corporate finance models that do not take into account liquidity risk. In banking, most models focus on the risk of illiquid asset returns, loans in my model. Here we study a specific type of risk that captures how liquidity provisions are covarying with asset returns, and we find that this risk – aggregate asset risk – increases with capital ratio.

Proposition 1. *An increase in capital requirements* increases *aggregate asset risk.*

Increased capital requirements lowers liquidity risk by decreasing the size of the liquidity shocks. This creates two channels leading to an increase in aggregate asset risk. The first is a decrease in firesale costs in both the high and the low liquidity shock states. Firesales are therefore less costly and the need to diversify the portfolio becomes less important in the low shock state. Risk-shifting incentives, however are still present in the high shock state due to limited liability. The second effect is an increase in loan investment. Because shocks are smaller, the need for liquid assets decreases and the bank invests more in long-term assets. However, an increase in loans increases the benefits of correlation. Because the bank holds less liquidity in the high liquidity shock state, it has to sell more illiquid assets, and a way to mitigate this effect is for these assets to be worth more. This can be done by increasing the correlation. In the low shock state, the mix of assets is irrelevant because firesales only occur when securities do not provide liquidity, hence it is as if the bank does not hold any.

Another way to interpret this result is to keep in mind the inverse relationship between securities holding and aggregate asset risk-taking. A capital increase lowers liquidity shocks and decreases securities holdings. As a result, it increases aggregate asset risk. Figure 4 illustrates this result.

4.2 Liquidity Regulations

Liquidity Coverage Ratio (LCR) is a recent tool in banking regulation that has been introduced as part of the Basel III framework. It aims at controlling the amount of liquid assets relative to deposits. In the context of the model, it means that the regulator can fix the ratio $\xi = \frac{\gamma}{1-k}$.

Introducing liquidity requirements simplifies the model, as it renders the mix of assest exogeneous. The bank's choice of aggregate asset risk is thus only governed by equation 8, the first order condition for ρ. We find that liquidity requirements have a positive effect on aggregate asset risk taking.

Proposition 2. *An increase in the Liquidity Coverage Ratio* decreases *aggregate asset risk taking.*

We find that an increase in the LCR has a positive impact on aggregate asset risk. Asking the bank to hold more securities has a beneficial impact on firesales in the high shock state. Because the bank does not need to rely as much on assets sales, it becomes costly to hold correlated assets if the low shock state realizes relative to the benefits in the high shock state. That gives incentives for decreasing the overall correlation. In addition, by limiting investment in illiquid assets, the bank reduces the profits of surviving the high shock state, which tilts even more the bank's choice towards uncorrelated assets.

Both capital and liquidity requirements aim at reducing the bank's default probability. However they do so in fundamentally different ways that impact aggregate risk-shifting incentives. Capital requirements affect all future states of the world and reduce all future liquidity shocks equally. It turns out that it has an adverse effect on aggregate asset risk. However, liquidity requirements are lowering liquidity shocks only in states where they are particularly severe. As a result, it is a much more effective tool for mitigating aggregate asset risk.

Figure 4 also suggests that the effect of liquidity requirements are altered by leverage. That is, the impact of liquidity requirements are different depending on the leverage. This is a very important implication for regulators. It means that what matters is the *joint* choice of capital and liquidity requirements, and this model highlights the interactions between both regulatory tools.

Proposition 3. *The impact of liquidity requirements on aggregate asset risk is higher for more levered banks.*

When leverage is high (small k), liquidity risk is at its highest as shocks are large. Consequently, firesales costs are more sensitive to changes in expected loans sales, leading to a greater impact of liquidity requirements on aggregate risk shifting.

4.3 Impact of liquidity shock

Finally, we try to explore how the characteristics of the liquidity shock might impact bank's liquidity management.

The model features aggregate uncertainty in the economy and one representative bank. That construction implicitely assumes that idiosyncratic risk is netted out and that the bank only faces systemic risk. However, the banking system is not as simple, and there is heterogeneity in the type of shocks faced by individual banks. For instance, it is likely that small banks face higher idiosyncratic risk than bigger banks. It might therefore be important to understand what the model has to say on any cross-sectional implications of liquidity management and aggregate risk taking.

I do so here in a very simple and reduced form way, by assuming that idiosyncratic risk influences the distribution of the liquidity shock.[9] More specifically, we assume that idiosyncratic risk increases the volatility of the liquidity shock. We find that small banks are more likely to hold more liquid assets but also to take on higher level of aggregate asset risk.

Proposition 4. *Banks that are facing more idiosyncratic risk hold more liquid assets but have higher aggregate asset risk profiles.*

When the high shock state becomes more prevalent, firesales losses are more likely to occur and holding liquidity is benefitial. However, if the large liquidity shock is more probable, limited liability increases risk shifting incentives, leading to a higher choice of correlation.

This result is interesting in the light of Basel 3 applicability of the LCR requirements in the United States. Liquidity requirements in the US only apply to banks whose total assets are larger than $250 billions, mainly large banks. However, a quick look at the FIDC insured bank's assets as of 2012 reveals that more than 40% of total banking assets are held by banks not subject to liquidity requirements. Proposition 4 highlights the need

[9] It is also quite intuitive to expect that firesales differ substantially when a systemic or idiosyncratic shock hits. I do not try to capture such effects here as it would require a much deeper analysis than what the current model can provide.

for increasing regulatory focus on small banks that have traditionally been ignored, as regulators around the world have been more concerned about systemic risk.

I have shown that capital ratio regulations tend to increase aggregate risk-taking while liquidity regulations tend to decrease is. In addition, the effectiveness of liquidity requirements is higher when banks are more levered. We have also highlighted the fact that small banks may be more affected by the mechanisms revealed in this paper, suggesting that imposing only capital regulations on small banks may not be enough if one is concerned about aggregate risk shifting.

5 Conclusion

In this paper, we propose a banking model of optimal portfolio choice combining illiquid assets and liquid securities. We depart from traditional banking models by assuming that bank securities provide stochastic future liquidity. Introducing uncertainty in future liquidity raises the question of how banks manage the risk between liquidity provisions and long term asset returns. This risk is defined as being the correlation between illiquid asset returns and liquidity provisions from securities, and is denoted *aggregate asset risk*. The presence of liquidity risk in the model generates risk-loving incentives that are balanced by diversification motives created by firesales. The model reveals that this trade-off gives rise to an inverse relationship between security holdings and aggregate risk taking. In other words, the more liquidities the bank holds, the less correlated its securities and long-term loans are. However, the bank tends to correlate its assets when it has more capital. It naturally leads me to show that current banking regulatory tools such as capital requirements and liquidity requirements have opposite effect on bank's appetite for aggregate asset risk. While imposing high capital ratio leads to higher aggregate risk taking, liquidity requirements are effective in decreasing risk-taking. The reason is that capital ratio affects all future liquidity shocks in an identical manner while liquidity requirements have the ability to dampen liquidity shocks when they are the most severe. Finally, we draw the cross-sectional prediction that small banks are potentially more subject to take on aggregate asset risk due to their increased exposure to idiosyncratic risk. Overall, these results show that there is a tension between capital and liquidity requirements. That is, both have opposite effects on aggregate risk taking. It therefore calls for cautious regulatory design, and suggests that a regulatory framework that imposes high capital ratio and high liquidity requirement may not be optimal if one is concerned about aggregate asset risk.

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Appendix

A Proofs

A.1 Liquidity Sock

A.1.1 Lemma 1

Here, we compute the thresholds λ_0, λ_1 and λ_2 of lemma 1. λ_0 is such that

$$\lambda_0 = \frac{\gamma r}{1-k}$$

λ_1 is such that

$$(1-\gamma) = F(M_0(\lambda_1)) \tag{9}$$

$$(1-\gamma) = F\left(\frac{\lambda_1(1-k)}{\frac{1}{2}(1-\rho)R}\right) \tag{10}$$

$$F^{-1}(1-\gamma) = \frac{\lambda_1(1-k)}{\frac{1}{2}(1-\rho)R} \tag{11}$$

$$\frac{1}{2}(1-\rho)RF^{-1}(1-\gamma) = \lambda_1(1-k) \tag{12}$$

$$\lambda_1 = \frac{\frac{1}{2}(1-\rho)RF^{-1}(1-\gamma)}{1-k} \tag{13}$$

λ_2 is such that

$$(1-\gamma) = F(M_r(\lambda_2)) \tag{14}$$

$$(1-\gamma) = F\left(\frac{\lambda_2(1-k)-\gamma r}{\frac{1}{2}(1+\rho)R}\right) \tag{15}$$

$$\frac{1}{2}(1+\rho)RF^{-1}(1-\gamma) = \lambda_2(1-k)-\gamma r \tag{16}$$

$$\frac{1}{2}(1+\rho)RF^{-1}(1-\gamma)+\gamma r = \lambda_2(1-k) \tag{17}$$

$$\lambda_2 = \frac{\frac{1}{2}(1+\rho)RF^{-1}(1-\gamma)+\gamma r}{1-k} \tag{18}$$

$\lambda_0 < \lambda_2$ is by construction and it is trivial to see that $\lambda_1 < \lambda_2$.

A.2 Bank's portfolio choice

A.2.1 Lemma 4

The bank optimizes the following program:

$$\begin{aligned}
\max_{\gamma,\rho} p_L & \left\{\gamma r + (1-\gamma)\frac{1}{2}(1+\rho)R - \lambda_L(1-k) + [(1-\gamma) - F(M_0^L)]\frac{1}{2}(1-\rho)R\right\} \\
& + p_H\left\{[(1-\gamma) - F(M_r^H)]\frac{1}{2}(1+\rho)R\right\} \\
& - p_L(1-\lambda_L)(1-k) - p_H(1-\lambda_H)(1-k) \\
& \text{s.t. } \underline{\gamma} < \gamma < \overline{\gamma}
\end{aligned}$$

Assuming that we have an interior solution, γ^* follows the FOC

$$\begin{aligned}
p_L(R-r) + p_H\left[\frac{1}{2}(1+\rho)R(1+\frac{\partial M_r^H}{\partial\gamma}F'(M_r^H))\right] &= 0 \\
p_L(R-r) + p_H\left[\frac{1}{2}(1+\rho)R(1-\frac{r}{\frac{1}{2}(1+\rho)R}F'(M_r^H))\right] &= 0 \\
p_L(R-r) + p_H\left[\frac{1}{2}(1+\rho)R - rF'(M_r^H)\right] &= 0
\end{aligned}$$

Solving for γ gives

$$\gamma^* = \overline{\gamma} - \frac{1}{2}(1+\rho^*)RF'^{-1}\left(\frac{1}{r}\left[\frac{1}{2}(1+\rho^*)R + (R-r)\frac{p_L}{p_H}\right]\right)$$

Which proves that $\gamma^* \leq \overline{\gamma}$.

ρ^* respects the FOC

$$\begin{aligned}
p_L\left\{\frac{1}{2}R(1-\gamma) - \left[\frac{1}{2}R[(1-\gamma) - F(M_0^L)] + \frac{\partial M_0^L}{\partial\rho}F'(M_0^L)\frac{1}{2}(1-\rho)R\right]\right\} & \\
+p_H\left\{\frac{1}{2}R[(1-\gamma) - F(M_r^H)] - \frac{\partial M_r^H}{\partial\rho}F'(M_r^H)\frac{1}{2}(1+\rho)R\right\} &= 0 \\
p_L\left\{F(M_0^L) - \frac{\partial M_0^L}{\partial\rho}F'(M_0^L)(1-\rho)\right\} + p_H\left\{(1-\gamma) - F(M_r^H) - \frac{\partial M_r^H}{\partial\rho}F'(M_r^H)(1+\rho)\right\} &= 0
\end{aligned}$$

We have

$$\frac{\partial M_0^L}{\partial \rho}(1-\rho) = M_0^L \tag{19}$$

$$\frac{\partial M_r^H}{\partial \rho}(1+\rho) = M_r^H \tag{20}$$

Pluging it into the FOC gives

$$p_H(1-\gamma) + p_H[M_r^H F'(M_r^H) - F(M_r^H)] - p_L[M_0^L F'(M_0^L) - F(M_0^L)] = 0$$

Expressing the price impact loss as $L(x) = xF'(x) - F(x)$, we have

$$p_H(1-\gamma) + p_H L(M_r^H) - p_L L(M_0^L) = 0$$

A.2.2 Lemma 5

Here we show that $\frac{\partial \gamma^*}{\partial \rho^*} < 0$ and that $\frac{\partial \rho^*}{\partial \gamma^*} < 0$. γ^* and ρ^* are jointly governed by equations 7 and 8. We define

$$g(\gamma^*, \rho^*) = \max\left[\underline{\gamma}, \overline{\gamma} - \frac{1}{2r}(1+\rho^*)RF'^{-1}\left(\frac{1}{r}\left[\frac{1}{2}(1+\rho^*)R + (R-r)\frac{p_L}{p_H}\right]\right)\right] - \gamma^* = 0 \tag{21}$$

$$h(\gamma^*, \rho^*) = p_H(1-\gamma^*) + p_H L(M_r^H) - p_L L(M_0^L) = 0 \tag{22}$$

We have that

$$\frac{\partial g}{\partial \gamma^*} < 0, \frac{\partial g}{\partial \rho^*} < 0 \tag{23}$$

$$\frac{\partial h}{\partial \gamma^*} < 0, \frac{\partial h}{\partial \rho^*} < 0 \tag{24}$$

Applying the implicit function, we obtain

$$\frac{\partial \gamma^*}{\partial \rho^*} = -\frac{\partial g/\partial \rho^*}{\partial g/\partial \gamma^*} < 0 \tag{25}$$

$$\frac{\partial \rho^*}{\partial \gamma^*} = -\frac{\partial h/\partial \gamma^*}{\partial h/\partial \rho^*} < 0 \tag{26}$$

$$\tag{27}$$

A.2.3 Lemma 3

We first prove that $\lambda_H < \lambda_2$ by showing that when $\lambda_H > \lambda_2$, $\frac{\partial V}{\partial \rho} > 0$. If this is true, λ_2 has to be such that $\lambda_2 \geq \lambda_H$. If $\lambda_H > \lambda_2$, the bank always defaults in the high shock state and its value function is

$$V = p_L \left\{ \gamma r + (1-\gamma)\frac{1}{2}(1+\rho)R - \lambda_L(1-k) + [(1-\gamma) - F(M_0)]\frac{1}{2}(1-\rho)R \right\}$$

So we have

$$\frac{\partial V}{\partial \rho} = \frac{\partial M_0}{\partial \rho}\frac{1}{2}RF'(M_0) > 0$$

We now prove that $\lambda_1 < \lambda_H$ similarly by showing that when $\lambda_H < \lambda_1$, $\frac{\partial V}{\partial \gamma} > 0$. If true, it has to be that $\lambda_1 \leq \lambda_H$. If $\lambda_H < \lambda_1$, the bank never defaults in the high shock state and its value function is

$$V = p_L \left\{ \gamma r + (1-\gamma)\frac{1}{2}(1+\rho)R - \lambda_L(1-k) + [(1-\gamma) - F(M_0^L)]\frac{1}{2}(1-\rho)R \right\} \tag{28}$$

$$+ p_H \left\{ [(1-\gamma) - F(M_r^H)]\frac{1}{2}(1+\rho)R + [(1-\gamma) - F(M_0^H)]\frac{1}{2}(1-\rho)R \right\} \tag{29}$$

So we have

$$\frac{\partial V}{\partial \rho} = \frac{\partial M_0^L}{\partial \rho}\frac{1}{2}RF'(M_0^L)p_L - \frac{\partial M_r^H}{\partial \rho}\frac{1}{2}RF'(M_r^H)p_H + \frac{\partial M_0^H}{\partial \rho}\frac{1}{2}RF'(M_0^H)p_H > 0$$

Note that these results are independent of the simultaneous choice of γ and ρ and also hold when γ is exogeneous.

A.3 Regulatory Implications

A.3.1 Proposition 1

To show that aggregate asset risk increase with capital requirements, we need to show that $\frac{d\rho^*}{dk} > 0$. ρ^* follows

$$p_H(1-\gamma^*(k)) + p_H L(M_r^H(\gamma^*(k),\rho^*)) - p_L L(M_0^L(\gamma^*(k)) = h(k,\rho^*) = 0$$

We have

$$\frac{\partial \gamma^*}{\partial k} < 0, \frac{\partial M_r^H}{\partial \gamma^*} < 0, \frac{\partial M_0^* L}{\partial k} < 0$$

It follows that

$$\frac{\partial h}{\partial \rho^*} < 0 \tag{30}$$

$$\frac{\partial h}{\partial k} = -\frac{\partial \gamma^*}{\partial k} + p_L L'(M_r^H)\frac{\partial M_r^H}{\partial \gamma^*}\frac{\partial \gamma^*}{\partial k} - p_L L'(M_0^L)\frac{\partial M_0^* L}{\partial k} > 0 \tag{31}$$

Therefore

$$\frac{d\rho^*}{dk} = -\frac{\partial h/\partial k}{\partial h/\partial \rho^*} > 0$$

A.3.2 Proposition 2

We express the FOC for ρ with the replacement $\xi = \frac{\gamma}{1-k}$. ρ^* follows

$$h(\xi,\rho) = p_H(1-\xi(1-k)) + p_H L(M_r^H(\xi,\rho^*) - p_L L(M_0^L(\rho^*)) = 0$$

We have

$$\frac{\partial h}{\partial \xi} < 0 \tag{32}$$

$$\frac{\partial h}{\partial \rho *} < 0 \tag{33}$$

Applying the implicit function theorem gives

$$\frac{\partial \rho^*}{\partial \xi} = -\frac{\partial h/\partial \xi}{\partial h/\partial \rho*} < 0$$

A.3.3 Proposition 3

To show that the sensitivity of aggregate asset risk to a change in liquidity requirements is higher for more levered banks, we need to show that $\frac{\partial^2 \rho^*}{\partial \xi \partial k} > 0$ We have that

$$sign\left(\frac{\partial^2 \rho^*}{\partial \xi \partial k}\right) = sign(1 + \frac{\partial^2 M_r^H}{\partial \xi \partial k}) \tag{34}$$

$$+ p_H \left(\frac{\partial M_r^H}{\partial k} L''(M_r^H) \frac{\partial M_r^H}{\partial \rho^*} + \frac{\partial^2 M_r^H}{\partial \rho^* \partial k} L'(M_r^H)\right) \tag{35}$$

$$- p_H \left(\frac{\partial M_0^L}{\partial k} L''(M_0^L) \frac{\partial M_0^L}{\partial \rho^*} + \frac{\partial^2 M_0^L}{\partial \rho^* \partial k} L'(M_0^L)\right) \tag{36}$$

It can then easily be verified that

$$\frac{\partial^2 \rho^*}{\partial \xi \partial k} > 0$$

A.3.4 Proposition 4

From equations 7 and 8, it is quite trivial to see that

$$\frac{\partial \rho}{\partial p_H} > 0$$
$$\frac{\partial \gamma}{\partial p_H} < 0$$